Date : 03/11/2015   Dept. No.   Max. : 100 Marks
Time : 01:00-04:00

Part A

Answer all questions   **10 x 2 =20 marks**

1. Find whether the following force is conservative

   $\vec{F} = (ax + by^2)\,\hat{\imath} + (az + 2bxy)\,\hat{\jmath} + (ay + bz^2)\,\hat{k}$ where a and b are constants.

2. What are generalized coordinates? If a generalized coordinate has the dimension of momentum, what would be the dimension of generalized velocity?

3. What are moments of inertia and products of inertia?

4. Express the rotational kinetic energy of a body in terms of inertia tensor and angular velocity.

5. Determine the generalized momenta, given the Lagrangian

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{V}{r}$$

6. Determine $[J_x, x]$.

7. What is the nature of the path traced by a representative point in a two dimensional phase space for a one dimensional harmonic oscillator?

8. What is the nature of the new set of variables (P_k, Q_k) when there is a canonical transformation from the set of variables (p_k, q_k) to (P_k, Q_k) and the transformed Hamiltonian is zero?

9. What are coupled oscillators?

10. Explain stable and unstable equilibrium.

Part B

Answer any FOUR questions   **4 x 7.5 = 30 marks**

11. Obtain the equations of motion of a simple pendulum by using Lagrangian method and hence deduce the formula for its time period for small amplitude oscillations.

12. Derive an expression for the rotational kinetic energy of a rigid body.

13. Show that the shortest distance between two points in a plane is a straight line using variational principle.

14. Discuss in detail the Hamilton Jacobi theory.

15. Obtain the eigenvalues of the two coupled pendulum using the theory of small oscillations.

16. Write a note on Infinitesimal contact transformations.

Part C

Answer any FOUR questions **4 x 12.5 = 50 marks**

17. Obtain the Lagrangian for the charged particle moving in an electromagnetic field.

18. Define Euler's angles and obtain an expression for the complete transformation matrix.

19. Explain Legendre transformation. Discuss how the transformation equations can be obtained from the generating functions of type F_1 and F_2.

20. What are action angle variables? Explain how they can be used to obtain the frequencies of periodic motion.

21. Discuss the double pendulum and obtain its eigenvalues using the theory of small oscillations.

22. What are Kepler's laws of planetary motion? Give the proof of Kepler's laws of planetary motion.